4.7 Identity and Inverse Matrices

Identity Matrix--square matrix that when multiplied by another matrix, it equals that same matrix.

$A \cdot I = A \qquad I \cdot A = A$

$\underset{2x2}{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \underset{3x3}{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Inverse matrices--are 2 square matrices whose product is the identity

A^{-1} -- "A inverse"

$A \cdot A^{-1} = I$

$A^{-1} \cdot A = I$

Are they inverses? yes

$\begin{bmatrix} 1,1 & 1,2 \\ 2,1 & 2,2 \end{bmatrix}$

$\underset{2x2}{A} = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix} \qquad \underset{2x2}{B} = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$

$A \cdot B \overset{?}{=} I$

$B \cdot A \overset{?}{=} I$

$AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$BA = \begin{bmatrix} 3(1)+2(-1) & 1(-2)+2(1) \\ 3(1)+3(-1) & 1(-2)+3(1) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Are they inverses?

$C = \begin{bmatrix} 3 & -1 \\ 4 & -2 \end{bmatrix} \qquad D = \begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix}$

$CD \overset{?}{=} I$

NO

Finding the inverse.

If D = 0, there is no inverse.

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad A^{-1} = \frac{1}{D}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$A = \begin{bmatrix} 6 & 4 \\ -1 & 3 \end{bmatrix} \begin{matrix} -4 \\ \\ 18 \end{matrix}$$

$D = 22$

$$A^{-1} = \frac{1}{22}\begin{bmatrix} 3 & -4 \\ 1 & 6 \end{bmatrix} = \begin{bmatrix} 3/22 & -2/11 \\ 1/22 & 3/11 \end{bmatrix}$$

HW
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